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AUGMENTATION OF ROCKET PROPULSION: PHYSICAL LIMITS

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INTRODUCTION

Consider a vehicle of initial mass m_0 moving at velocity v_0 through a medium which is initially at rest. One dimensional motion will be assumed. The vehicle initially has stored energy E_s. At a later time, the stored energy has all been used, and the vehicle has mass m_p and velocity v_p. The vehicle has interacted with a total mass m_a of the medium. The vehicle exhaust of mass

$$m_r = m_0 - m_p$$

has been expelled into the medium. Conservation of momentum and energy lead to the following equations.

$$m_0 v_0 = m_p v_p - (m_a + m_r)(v_r)_{av}$$

$$E_s + \tfrac{1}{2} m_0 v_0^2 = \tfrac{1}{2} m_p v_p^2 + \tfrac{1}{2}(m_a + m_r)\left(v_r^2\right)_{av}$$

$(v_r)_{av}$ is the average velocity of the medium with which the vehicle interacts together with the expelled exhaust products, taken to be in the opposite direction from v_0 and v_p. The total mass of medium with which the vehicle interacts is m_a. The two equations can be combined to give the following equation for the variance of the air velocity distribution.

$$\sigma^2 = \left(v_r^2\right)_{av} - (v_r)_{av}^2 = \frac{-m_p(m_0 + m_a)v_p^2 + 2m_p m_0 v_p v_0 + m_0(m_p + m_a)v_0^2 + 2E_s(m_r + m_a)}{(m_r + m_a)^2}$$

The equation above can be rearranged as an equation for the velocity increase.

$$v_p - v_0 = \frac{-m_a v_0 + \sqrt{m_0 v_0^2 + \frac{(m_0 + m_a)}{m_p}\left[2(m_r + m_a)E_s - m_0(m_p - m_a)v_0^2 - \sigma^2(m_r + m_a)^2\right]}}{m_0 + m_a}$$

The vehicle velocity increase is greatest for the smallest variance in the velocity imparted to the medium. In this case, we have

$$\frac{v_p - v_0}{u} = \frac{\sqrt{(\mu + \mu_a - 1)\left(\mu + \mu_a + \mu\mu_a \frac{v_0^2}{u^2}\right)} - \mu \frac{v_0}{u}}{\mu + \mu_a}$$

where

$$\mu = \frac{m_0}{m_p} \qquad \text{and} \qquad \mu_a = \frac{m_a}{m_p}$$

Even if we assume σ^2=0, the momentum transfer is not well defined until a model is specified for the mass of the interacting medium. It is instructive to consider a limiting case before proceeding to a more realistic model. In the limit $m_a \to \infty$ we have the following.

$$\frac{v_p - v_0}{u} = \sqrt{\mu \frac{v_0^2}{u^2} + 1} - \frac{v_0}{u}$$

This limit is appropriate if a vehicle pushes against an entire planet. It would apply to an automobile, for example (assuming the automobile carries its own oxygen and the exhaust is left at rest with respect to the road). For v_0=0, the right-hand side approaches unity, which means that all of the initial stored energy becomes payload kinetic energy.

The model above is not realistic because of the assumption σ=0. In order to control σ, one would need a means of storing some of the energy when fuel is burned in order to use it at a later time. More realistically, the energy must be used at the time the fuel is burned. Despite this, the calculation correctly shows that a performance benefit can be realized when a rocket pushes against the medium through which it travels.

OPTIMAL PROPULSION IN A MEDIUM

I now construct a more realistic model in which the energy from the fuel is used immediately to accelerate air and propellant. This implies that

$$dE = \gamma dm$$

where γ is the specific energy of the fuel. Also, the interacting air mass in any time interval is proportional to the volume swept out by the vehicle in the same interval.

$$dm = \rho A v dt$$

Conservation of momentum and energy in any time interval can be written as follows.

$$mv = (m + dm)(v + dv) - (dm_a - dm)v_r$$

$$E + \tfrac{1}{2}mv^2 = E + dE + \tfrac{1}{2}(m + dm)(v + dv)^2 + \tfrac{1}{2}(dm_a - dm)v_r^2$$

The quantities dm and dE, the changes in mass and stored energy of the vehicle in a time interval, are both negative. The density of the medium is ρ and the cross-sectional area of the medium swept up for propulsion is A . The acceleration can be used to eliminate the time increment.

$$dt = \frac{dv}{a}$$

The reaction speed v_r can be eliminated to give the following differential equation for the velocity change as a function of the mass change.

$$dv = \frac{-2\gamma dm}{\sqrt{2\gamma m\left(m + \frac{2\rho A v^2}{a}\right) + \left(\frac{\rho A v}{2a}\right)^2 \left(2\gamma + v^2\right)^2} - \left(\frac{\rho A v}{2a}\right)\left(2\gamma + v^2\right)}$$

The equation has been integrated numerically for constant A. ρ, and a to give the curves of Figure 1.

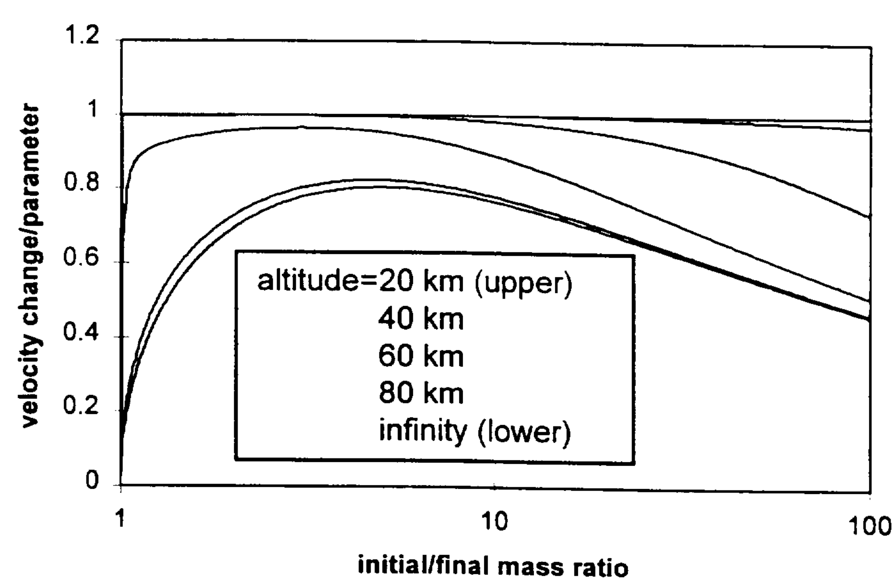

Figure 1. Performance of rocket augmented by pushing on bypassing air.

The four curves of Figure 1 apply to atmospheric densities typical of altitudes of 20, 40, 60, and 80 km, for a cross-sectional area, payload mass, and specific energy typical of the space shuttle with the external tank attached. At the higher densities, the performance graphs are near unity. Even at an air density representative of 60 km, the optimal air pusher has a large performance advantage over a conventional rocket for vehicles with a high initial to payload mass ratio. When the vehicle is used at 80 km, however, the advantage of air pushing is very little.

OPTIMAL MEDIUM INTAKE

We have seen that the physical performance limits are higher for a vehicle which pushes on its medium than for a completely closed system, particularly at low initial to final mass ratios. The analysis so far has not included drag. It will now be shown that when drag is included, there is a well-defined intake size for optimum performance.

With drag, the momentum and energy equations are as follows.

$$mv = (m + dm)(v + dv) - (\rho A_a v dt - dm)v_r + \rho A_d v^2 dt$$

$$E + \tfrac{1}{2} mv^2 = E + dE + \tfrac{1}{2}(m + dm)(v + dv)^2 + \tfrac{1}{2}(\rho A_a v^2 dt - dm)v_r^2 + \rho A_d v^3 dt$$

The drag area A_d is usually related to the frontal area A with the drag coefficient. The effective intake area is A_a. The elimination of v_r can be done as before, which leaves the following relation among dm, dv, and dt.

$$0 = 2\gamma dm^2 - \rho A_a v dt(2\gamma + v^2)dm - m^2 dv^2 - \rho v^2 dt\left[2mdv(A_a + A_d) + \rho A_d v^2 dt(2A_a + A_d)\right]$$

Instead of eliminating dt at this point, I will consider flight at constant v. This enables the analysis of the optimum intake size at various speeds and air densities. The full treatment of accelerated motion should be in qualitative agreement, except that very large intakes will be discouraged by the mass penalty.

The rate of fuel consumption as a function of velocity for $v<u$ is approximately

$$\frac{dm}{dt} = \frac{-\rho A_d v^3 (2A_a + A_d)}{2\gamma A_a}$$

Note that the area factors are separate from the density and velocity, so that within this model the <u>intake size scales independently of the velocity and the density</u>. The fuel consumption rate can be written

$$\frac{dm}{dt} = \frac{-\rho v^3 F}{2\gamma}$$

where the geometric factor F is given by

$$F = \frac{A_d(2A_a + A_d)}{A_a}$$

At first glance it would appear that A_a should be made as large as possible, but this is not correct, since A_a is not independent of A_d. A simple model for the areas includes an irreducible area A_p associated with the payload. In addition, there is an intake which can be modeled as a thin annulus of radius r and width w. Then the geometric factor in the fuel consumption is

$$F = \frac{(A_p + 2\pi r w)(2\pi r^2 + A_p + 2\pi r w)}{\pi r^2}$$

As an example, suppose A_p=50 m^2 and w=1 m. Figure 2 shows the geometric factor F for this example. The optimal radius is 6.1 m. The intake area in this case is about 2.3 times the effective frontal area of the payload. When the mass is taken into account for an accelerating

vehicle, the optimal intake area will be somewhat smaller. Note that the minimum is such that the performance suffers only slightly if the radius is decreased to about 3m, but then the performance degrades severely for smaller intakes. For the smaller intake, the intake area is about 60 % of the effective area for drag.

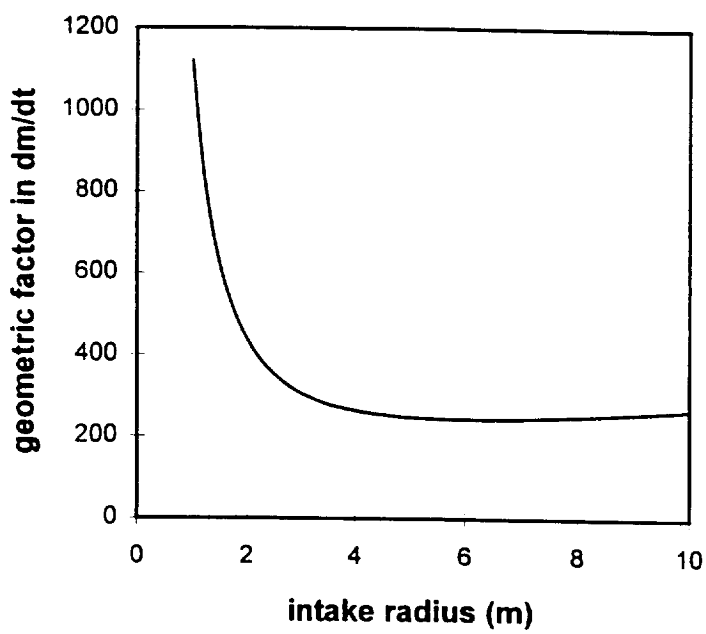

Figure 2. Geometric factor in fuel consumption for augmented rocket propulsion at constant velocity. The effective area for drag is taken to be 50 m. The intake is modeled as an annulus of thickness 1 m.

CONCLUSIONS

Rocket propulsion is not ideal when the propellant is not ejected at a unique velocity in an inertial frame. An ideal velocity distribution requires that the exhaust velocity vary linearly with the velocity of the vehicle in an inertial frame. It also requires that the velocity distribution variance as a thermodynamic quantity be minimized.

A rocket vehicle with an inert propellant is not optimal, because it does not take advantage of the propellant mass for energy storage. Nor is it logical to provide another energy storage device in order to realize variable exhaust velocity, because it would have to be partly unfilled at the beginning of the mission.

Performance is enhanced by pushing on the surrounding because it increases the reaction mass and decreases the reaction jet velocity. This decreases the fraction of the energy taken away by the propellant and increases the share taken by the payload. For an optimal model with the propellant used as fuel, the augmentation realized by pushing on air is greatest for vehicles with a low initial/final mass ratio. For a typical vehicle in the Earth's atmosphere, the augmentation is seen mainly at altitudes below about 80 km. When drag is taken into account, there is a well-defined optimum size for the air intake.

Pushing on air has the potential to increase the performance of rockets which pass through the atmosphere. This is apart from benefits derived from "air breathing", or using the oxygen in the atmosphere to reduce the mass of on-board oxidizer. Because of the potential of these measures, it is vital to model these effects more carefully and explore technology that may realize their advantages.